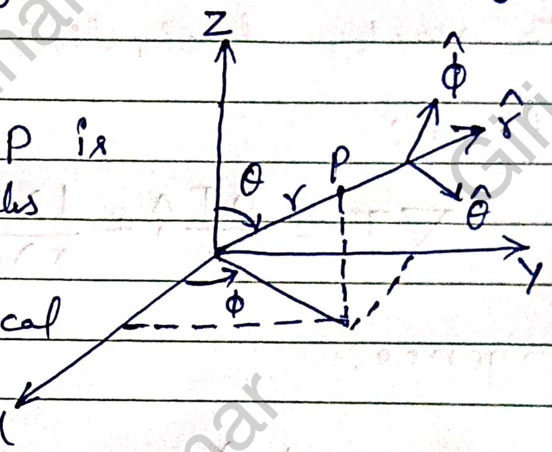
* Spherical and Cylindrical Coordinate Systems.

*** Spherical coordinate :—** Spherical coordinates (r, θ, ϕ) are a three-dimensional coordinate system used to describe points in space based on their distance and angles from a fixed origin. They are particularly useful in problems with spherical symmetry, such as in physics and engineering.



in given figure a point P is with its Cartesian coordinates (x, y, z) but it is more convenient to use spherical coordinates (r, θ, ϕ) where
r = the distance from the origin.

θ = the angle down from the z axis or ~~pl~~ polar angle.

ϕ = It is the azimuthal angle.

The relation to Cartesian coordinates can be read from.

$$\left.\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}\right\} \text{—— (1)}$$

In given figure, the three unit vectors, $\hat{r}, \hat{\theta}, \hat{\phi}$ pointing in the direction of increase of the ~~Correspoind~~ corresponding coordinate. They constitute an orthogonal basis set $\hat{x}, \hat{y}$ and $\hat{z}$

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and any vector A can be expressed in terms of them in the usual way:.

$$A = A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}. \quad \text{——— (ii)}$$

Where A_r, A_θ and A_ϕ are the radial, polar and azimuthal components of A. In terms of the cartesian unit vectors,

$$\left.\begin{aligned} \hat{r} &= \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z} \\ \hat{\theta} &= \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z} \\ \hat{\phi} &= -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} \end{aligned}\right\} \quad \text{——— (iii)}$$

* The vector derivatives in spherical coordinates:.

Gradient:-

$$\nabla T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\cdot\frac{\partial T}{\partial \phi}\hat{\phi} \quad \text{——— (iv)}$$

Divergence:-

$$\nabla\cdot v = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\cdot\frac{\partial v_\phi}{\partial\phi} \quad \text{——— (v)}$$

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$$\nabla\times v = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\cdot\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi} \quad \text{——— (vi)}$$

Laplacian:-

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 T}{\partial\phi^2} \quad \text{——— (vii)}$$

* Q ① Find formulas for (r, θ, ϕ) in terms of x, y, z.

Q②

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